THE UNITED REPUBLIC OF TANZANIA
PRESIDENT'S OFFICE
REGIONAL ADMINISTRATION AND LOCAL GOVERNMENT

FORM SIX SPECIAL SCHOOLS SYNDICATE JOINT EXAMINATION

142/1 **ADVANCED MATHEMATICS 1**

Time: 3:00 Hrs **Friday 10-March-2023 PM**

INSTRUCTIONS

1.This paper consists of ten(10) questions each carrying ten(10) marks

2.Answer all questions.

3.All necessary working and answers for each question done must be shown clearly.

4.Non-programmable scientific calculators may be used.

5.Cellular phones and any unauthorized materials are not allowed in the examination room .

6.Write your Examination Number on every page of your answer booklet(s).

1. (a) Use a non-programmable calculator to evaluate the values of the following:

(i) $\sqrt[7]{\dfrac{\cot\left(e^{2}\right)+{}^{5}P_{2}\cosh(\ln 2)}{\sin 63^{0}21'+\sum_{r=1}^{2}\ln(5r)}}$ Correct to three decimal place

(ii) $\ln\left(\dfrac{\sqrt{98.2}\times(0.0076)^{-1}\times 10^{-2}}{\tan\dfrac{\pi}{3}\times\cos^{3}\dfrac{\pi}{4}}\right)$ Correct to four significant figures.

(b) The rate of population is directly proportional to the number of inhabitants present at that time, t is given by $N=N_0e^{kt}$, where N is the population at any time, t, N_0 is the original population and k is a constant. Consider the population growth in city X below:

Year	Population
1985	20,000,000
1995	24,000,000
2000	

By using your non programmable calculator, find the population in the year 2000.

2. (a) Solve the equation for exact value of x: $2\sinh x-\cosh x=2\tanh\dfrac{x}{2}$.

Leave your answer in logarithmic form.

(b) Prove that $16\sinh^{2}x\cosh^{3}x=\cosh 5x+\cosh 3x-2\cosh x.$

Hence or otherwise evaluate $\int_0^1 16\sinh^{2}x\cosh^{3}x\,dx$ giving your answer in terms of e.

3.A medical company has factories at two places P_1 and P_2 with production capacities 60 and 70 packets of medicine respectively.From these places ,supply is made to each of its three agencies situated at A_1 ,A_2 and A_3. The monthly requirements of the agencies,are respectively 40,40 and 50 packets.The transportation costs(TZS) per packet for the factories to the agencies are given below:

FROM / TO	P_1	P_2
A_1	5000	4000
A_2	4000	2000
A_3	3000	5000

(a)How many packets from each factory should be transported to each agency so that the cost of transportation is minimum?

(b)Find the minimum cost.

(c)Is the transportation balanced?why?

4.(a)Calculate the mean and standard deviation of the first n even numbers.

(b)If $\overline{x}$ $is\ the\ mean\ of\ x_1\ , x_2\ x_3\ \ldots, x_n$,show that the mean of $ax_1\ , ax_{2,} ax_3\ \ldots, ax_n$ is a$\overline{x}$.

(c)Given that the Var(X)= $\frac{1}{N}\sum_{i=1}^{n} \mathrm{f_i}(\mathrm{x_i}-\overline{\mathrm{x}})^2$,show that $\delta=\sqrt{\frac{\sum fd^2}{N}-\left(\frac{\sum fd}{N}\right)^2}$,

where $x_i = A + d_i$.

(d)Calculate the variance and standard deviation of the following weekly wages of workers in a certain commercial institution given below:

Weekly wages(Rs)	40-43	43-46	46-49	49-52	52-55
No.of workers	31	58	60	44	27

5.(a)Use the laws of algebra of sets to simplify the following sets:

(i)$[(A\cap B^c)\cup(A\cap B)\cup(A^c\cap B)\cup(A\cap B)^c]^c$

(ii)$(A\cap B^c)\cup(A^c\cup B)$

(b) 60 students study Economics, Geography and Mathematics. 12 students score A in both Economics and Geography. Half of the students score A in Mathematics, one third score A in Geography. 5% of the students score A in Economics only. If 17 students don't have A in any subject, how many score A in;

(i) All three subjects

(ii) Geography only

(c)) A is a set define as
A = $\{x^3 : x \in z^+ \text{ and } 0 \leq x < 5\}$. Describe set A by roster form

6.The function f(x) is defined by $f(x)=\begin{cases} x,\ 0<x<2 \\ x-1,\ 1\leq x\leq 2 \\ x-n, n\leq x\leq n+1\ , where\ n\varepsilon z \end{cases}$

(a) Sketch f(x) and find its domain and range

(b)What are the values of f (2.5) and f(3)?

(c)Sketch f(x) $=\frac{x+1}{(x+1)(x-1)}$

7.(a)Derive Newton Raphson formula from Taylor`s theorem.

(b)Using both Simpson and Trapezium rule with six ordinates,approximate the value of $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}d\theta$ correct to four decimal places.

(c) (i) Verify whether the equation $x^3+2x^2-5x-6 = 0$ has a root between x = 1.9 to 2.9

(ii) Use the N-R formula in three iterations to find the root.

8.(a)The straight line L passes through the point (a,3) where a is a constant and is perpendicular to the line with equation 3x+4y=12,given that L crosses the y-axis at (0,-5).Find the value of a.

(b)Prove that the line 2x-3y-27=0 is a tangent to the circle $x^2+y^2-8x+4y+7=0$.

(c) (a) The end point coordinates of a line segment PQ are $P(x_1, y_1)$ and $Q(x_2, y_2)$ Prove that the coordinates of the point A(x, y) dividing the line segment PQ in the ratio m:n internally is $A(x, y) = \left[\frac{nx_1+mx_2}{m+n}, \frac{ny_1+my_2}{m+n}\right]$

9.(a)By means of substitution $x=1-\frac{1}{u^4}$,show that $\int_5^{2.25} \frac{1}{u\sqrt{2u^4-1}} du = \frac{\pi}{24}$.

(b)Evaluate $\int_1^2 \frac{1}{x^2\sqrt{x-1}} dx$.

(c)Find $\int \frac{1}{1+sin2x} dx$.

10.(a) A tank in the form of an inverted cone having an altitude of 16m and base radius 4m long.Water is flowing into the tank at the rate of $2m^3$/min. How fast is the water level rising when the water is 5m deep?

(b)Differentiate with respect to x (i)2^{x^2} $(ii)x^{x\sin x}$.

(c)Expand $\sin(\frac{\pi}{6} + h)$ as far as h^5 and hence evaluate $\sin 35°$ correct to four decimal places.

(d)Find the area bounded by the curves $y^2=4x$ and $x^2=4y$.