TANZANIA HEADS OF ISLAMIC SCHOOLS COUNCIL

FORM SIX INTER ISLAMIC MOCK EXAMINATION

ADVANCED MATHEMATICS 1

142/1 **(For both School and Private Candidates)**

TIME: 3 HOURS **Tuesday, 07th March 2023 a.m.**

Instructions

1. This paper consists of ten (10) compulsory questions.
2. All work done in answering each question must be shown clearly.
3. Mathematical tables and non-programmable calculators may be used.
4. Cellular phone and any unauthorized materials are not allowed in the examination room.
5. Write your Examination Number on every page of your answer booklet(s)

1. Use a non-programmable calculator:

(a) Find each of the following:

(i) $\frac{Co\,sec^{-1}\,Sin\frac{\pi}{2}+log_5(\sqrt{x}\times 10^{-3})}{ln\,2+tan^{-1}\,tan\,30^o21'41''}$

Correct to 6 decimal places.

(ii) $\sum_{x=2}^{4} e^{\ln x}\left[(2+(1+\ln x).\ln x\right]$

(b) Find the mean and standard deviation of the frequency distribution below:

Score	30.5 - 44	44.5 - 58	58.5 - 72	72.5 - 86	86.5 - 100
No. of students	9	12	8	10	6

(c) If $A=\begin{pmatrix} 2 & 3 & -1 \\ 2 & 0 & 8 \\ 2 & 4 & 5 \end{pmatrix}$ and $B=\begin{pmatrix} 7 & 6 & -1 \\ 0 & 0 & 8 \\ 2 & 4 & 3 \end{pmatrix}$

Find 2 det A + det B.

2. (a) Express the function f(x) = 5coshx + 3sinhx in the form of R cosh (x+α). Hence find minimum value of f(x) and the value of x for which the minimum value occurs.

(b) Define coshx and sinhx and hence prove that

$\frac{1}{\cosh 2x+\sinh 2x}\equiv \cosh 2x-\sinh 2x.$ Hence or otherwise show that

$$\int_0^1 \frac{dx}{cosh\,2\,x + sinh\,2\,x} = \frac{1}{2}(1 - e^{-2})$$

(c) Solve the equations

$$\begin{cases} \cosh x - 3\sinh\ y = 0 \\ 2\sinh\ x + 6\cosh y = 5 \end{cases}$$

giving answers in logarithmic form.

(d) Hence or otherwise, show that

$$\int_0^1 \frac{dx}{\cosh 2x+\sinh 2x} = \frac{1}{2}\left(1-e^{-2}\right)$$

3. (a) Write down four steps of formulating linear programming problem.

(b) A sugar company ships sugar from two origins S_1 and S_2 to three market centres M_1, M_2 and M_3. The table below shows the available tons of sugar and the required tons together with the unit transportation cost in shillings.

	M_1	M_2	M_3	Available
S_1	20	10	5	220
S_2	10	25	30	100
Requirement	120	80	120	

(i) Formulate the objective function using the information given.

(ii) Write down all inequalities of the transportation problem.

(iii) Verify whether the transportation problem above is balanced or not.

(iv) Find the minimum cost.

4. (a) The mean and variance of 7 observations are 8 and 16 respectively.

If observations are 2, 4, 10, 12 and 14. Find the remaining two observations.

(b) The scores of the mathematics test conducted at MTAKUJA Secondary school are summarized in the table below:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of student	4	7	10	9	11	19	21	8	5	2

Determine:

(i) The mean and variance by coding method.

(ii) The 60^{th} percentiles of the score.

(iii) The interquartile range of the score.

(c) If there are only two measurements m and n. Show that variance = $\frac{R^2}{4}$, where R=range.

5. (a) Use laws of algebra of sets to simplify

$(A' \cap B' \cap C) \cup (B \cap C) \cup (A \cap C)$.

(b) Without using Venn diagram prove that:

n(AuBuC) = n(A)+n(B) + n(C)-n(AnB) – n(AnC) – n(BnC) +n(AnBnC).

(c) A class contains 15 boys and 15 girls. If 20 students takes science, 14 students takes Mathematics, of boys 10 takes science and 10 takes Mathematics, 8 of the boys takes both Science and Mathematics. Find the number of the students in the class who take:

(i) Both science and mathematics

(ii) Mathematics but not science.

(iii) Exactly one subject.

6. (a) Given that $x = \{1,2,3,4,5\}$ and $g(x) = \{(1,0),(2,1),(3,3),(4,6),(5,8)\}$ and $f(x)=2x-4$. Find:

(i) $fog(x)$

(ii) $gof(x)$

(b) f is a function defined by $f(x)=2^{x-1}-2$. Find:

(i) The domain and range of f.

(ii) The horizontal asymptote of f.

(iii) The x and y – intercepts.

(iv) Sketch the graph of f.

7. (a) Use the Newton – Raphson method

(i) To find the formula of finding the reciprocal of number R.

(ii) Use the derived formula above to find $\frac{1}{x} = 1.37$. Take x_o= 0.75.

(b) Use both Simpson's rule and Trapezium rule with 5 ordinates to estimate the value of

$\int_0^1 \cos\sqrt{x}\ dx$.

8. (a) Determine the ratio in which the line 3x + 4y – 9=0 divides the line segment joining points A (1, 3) and B (2, 7).

   (b) Find the equation of locus of points 3 units away from the midpoint of (3, 1) and (1, 1).

   (c) Find the area of the triangle made by points of contact of tangents drawn from A(15, 12) to the circle $x^2+y^2 - 4y - 10x + 9 = 0$.

9. (a) Evaluate $\int \frac{x^2+1}{(x+1)(x-1)}dx$.

   (b) If $\int_0^{\frac{1}{2}} \frac{dx}{\sqrt{x-x^2}} = \frac{\pi}{k}$. Find the value of k.

   (c) The area of the line segment cut off by y = 3 from the curve $y= x^2-1$ is rotated about y=3. Find the volume generated.

10. (a) Differentiate $f(x) = 2x^2 + \text{Sin}2x$ from first principle.

   (b) If $x = 2at^2$ and y = 4 at. Find $\frac{d^2y}{dx^2}$ in terms of t.

   (c) A farmer has an adjustable electric fence that is 100m long. He uses this fence to enclose a rectangular grazing area on three sides. The fourth side being a fixed hedge. Find the maximum area he can enclose.

Wabillah Tawfiiq